The Low Power Helium Pulsed Arcjet
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Abstract

An electrothermal thruster that operates in a rapid-pulse mode at low power (<200 W) is investigated. The thruster, called a pulsed arcjet, uses a capacitor and a pulse-forming electrical circuit to transfer stored electrical energy to a propellant gas in 3-10 µsec arc discharges at repetition rates of 550 to 2600 pulses-per-second with pulse energies from 24 to 130 mJ. The arc discharges occur in a cylindrical capillary upstream of a converging-diverging nozzle, and all the energy addition occurs in the subsonic region. Peak currents in the arc are 110 to 270 amps. Performance is measured using helium propellant for two 20-degree half angle conical nozzles with area ratios of 20 and 230. Thrust levels from 10 to 30 µN for power levels of 24 to 119 watts are measured on an inverted pendulum-type thrust stand with input power levels determined from measurements of pulse rate and breakdown voltage. A maximum specific impulse of 305 seconds is achieved with 38% efficiency. A time-dependent, quasi-1D numerical model is developed to evaluate energy losses using a time-marching procedure. Viscous and heat transfer effects are incorporated through a friction factor and an average heat transfer coefficient. The specific impulse and efficiency are found to be sensitive to wall temperature due to heat transfer losses in the subsonic region. Viscous effects become important as the specific energy increases above 12 MJ/kg and the throat Reynolds number falls below 1000. A pulse-forming electrical circuit is employed which utilizes high current diodes across the capacitor terminals to eliminate current reversals. Greater than 85% of the initial stored energy is transferred to the arc in a unipolar pulse.

Introduction

The origins of the pulsed arcjet come from work performed by Burton on the pulsed electrothermal thruster (PET) in the 1980's.1 The development of the PET involved liquid water propellant with its associated two-phase flow problems, and one conclusion of the experimental work was that a switch to a gaseous propellant might be beneficial. While the PET thruster operated at low frequency with high pulse energies, the pulsed arcjet operates at high frequency with low pulse energies. Preliminary experiments with the pulsed arcjet were conducted using simulated-hydrazine propellant; however, the performance did not appear promising, and the range of test conditions was restricted by the unfavorable breakdown characteristics of the diatomic propellant. The performance and operating characteristics greatly improved with helium propellant and allowed clear performance trends to be obtained experimentally.

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List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>f</td>
<td>Friction factor</td>
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<tr>
<td>h</td>
<td>Enthalpy [J/kg]</td>
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<td>I</td>
<td>Arc current [amps]</td>
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<td>ISP</td>
<td>Specific impulse [s]</td>
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<td>L</td>
<td>Inductance [H]</td>
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<td>m</td>
<td>Propellant mass [kg]</td>
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<td>Mass flow rate [kg/sec]</td>
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<td>Pulse rate (pulses per second) [Hz]</td>
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<td>Q</td>
<td>Heat transfer rate [W]</td>
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<td>R</td>
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<td>r</td>
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<td>r0</td>
<td>Arc channel radius [m]</td>
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<td>u</td>
<td>Flow velocity [m/s]</td>
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<td>Voltage [volts]</td>
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<td>v</td>
<td>Velocity [m/s]</td>
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<td>η</td>
<td>Thrust efficiency [%]</td>
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<td>κ</td>
<td>Thermal conductivity [W/m-K]</td>
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<td>ρ</td>
<td>Density [kg/m³]</td>
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<td>σ</td>
<td>Electrical conductivity [ohm-m]</td>
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The arc discharge is produced using a constant current power supply, a storage capacitor, and a pulse-forming electrical circuit directly coupled to the electrodes. The power supply ramps up the voltage and stored energy on the capacitor until the propellant breakdown voltage is reached, initiating an arc discharge in a cylindrical capillary between the electrodes. The energy stored on the capacitor is transferred to the propellant gas in a short intense pulse. The arc attaches to the nozzle upstream of the convergent section, and the heat addition to the gas is entirely subsonic. The arc completely extinguishes when the voltage on the capacitors is not sufficient to maintain the arc, and the charging cycle repeats. Utilizing the self-breakdown of the propellant eliminates the need for a switch or spark igniter, greatly simplifying the electrical circuit. The time between pulses is on the order of 10^-3 seconds, which is much longer than the arc discharge time of 10^-6 to 10^-8 seconds, so that the arc is present for only a small fraction of the total cycle time. Arc peak currents are several hundred amps, and arc voltages are typically 100 to 200 volts. Pulse energies are 20 to 130 mJ.

Experimental Apparatus

Mechanical Design

The pulsed arcjet assembly consists of the thruster itself and the pulse-forming electrical circuit including the storage capacitor. The arcjet design is based on a NASA 1-kW laboratory-type arcjet with modifications to the thruster head for pulsed operation. The thruster head is modified to include a cylindrical insulator tube called a capillary between the cathode and the anode/nozzle (Fig. 1). The anode is a 20-degree half-angle conical nozzle fabricated from 2% thoriated tungsten. The nozzle throat was machined using an EDM process to 0.38 mm diameter, measured from a photomicrograph. Two different nozzles with area ratios of 2 and 23 were fabricated to evaluate the tradeoff between the increased gas expansion at higher area ratio compared to viscous losses. Friction effects are significant because the Reynolds numbers in the pulsed arcjet are relatively low, ~1000. Capillary lengths from 5.0 to 12.5 mm were selected so that electrical breakdown would occur at voltages from 1000 to 2000 volts for the desired range of mass flow rates. The 2.5 mm capillary diameter was selected as a baseline geometry from volume considerations.

Propellant gas is injected axially into the capillary through a 0.28 mm diameter orifice in the cathode. The size of the orifice is smaller than the 0.38 mm nozzle throat diameter in order to limit the amount of gas that is blown back upstream after each arc discharge. The propellant flow is continuous and is not pulsed with a propellant valve. Because the pulsed arcjet is a thermal device, the surface temperatures of the nozzle and front thruster body have an important effect on overall performance. For the pulsed arcjet running on less than 150 watts in a 1-kW arcjet body, the steady state nozzle temperature is less than 800 K, much less than desired for optimal thruster performance.

Electrical Circuit

The pulse-forming electrical circuit was designed to provide non-reversing current to benefit capacitor life and to minimize energy losses in the external circuit. The shape of each current pulse, the peak current, and the duration of the arc are determined by the characteristics of the arc resistance and the design of the external electrical circuit. The circuit design (Fig. 2) includes a single 0.05 μF storage capacitor, a pair of high-voltage, high-current diodes, and a 2.9 μH inductor.
simply an underdamped LCR circuit. More complete details of the circuit components and the thruster mechanical design are described in previous work.\(^3\)

**Test Equipment**

The helium propellant is supplied at room temperature from a high-pressure gas cylinder with 99.995% purity. The propellant flows through a regulator valve to a Unit Instruments model UFC-1500A (5 slm) flow controller. The flow controller is calibrated to an accuracy of better than 2.7% using a control volume method. The thruster is operated in a 1.0 m diameter x 1.5 m long vacuum tank evacuated to approximately 50 mTorr with the thruster running.

Power is supplied to the capacitor through a high voltage coaxial power cable from a constant current, switching-type power supply (Converter Power Model RCS 3000), which provides a series of charging cycles to the storage capacitor. A separate control circuit produces zero charging current for a fixed time delay between pulses, allowing the pulse rate to be set externally and reducing pulse-rate instabilities caused by the feedback between the pulse rate and the breakdown voltage.

An inverted pendulum-type thrust stand is used for thrust measurements. A magnetic damper coil removes oscillations so that the steady state thrust can be measured. Since the pulse rate is two orders of magnitude greater than the thrust stand natural frequency, the output signal appears as a steady time-averaged thrust, and the individual impulse bits are not observed in the thrust measurement. Thrust calibration is performed before and after each test with a pulley arrangement providing three known force levels. A proportional-integral-derivative (PID) controller is used for feedback control.

For detailed measurements related to individual arc pulses or single charging cycles, a 4-channel, 8-bit, 10 MHz digital oscilloscope (Soltex) is used. Two inductive current monitors (Pearson models 411 and 4100) are used to measure the capacitor charging current and the arc discharge current. The capacitor charging voltage is measured using a low level 0-10 volt analog signal from the power supply. A 12-bit data acquisition card (Keithley Metrabyte DAS-1802HC) installed in a 486 personal computer is used to record four operating parameters: thrust, mass flow rate, breakdown voltage and pulse rate. The experimental uncertainties in thrust, power, specific impulse and efficiency are 1.9%, 4.1%, 3.3%, and 6.2%, respectively.

**Experimental Results**

For each performance test, the procedure is to allow the mass flow rate to reach steady state with room temperature gas and then apply power. In the first several seconds after power is applied, the thrust increases immediately with a proportional increase in I\(_{sp}\) of 10-60 seconds. The thrust and specific impulse then rise gradually as the arcjet heats up to steady-state temperature. Fig. 3 shows data from a typical run.

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model, which generally follow the experimental trends. The numerical results indicate that decreasing the mass flow rate from 10 mg/sec to 6.6 mg/sec at fixed power level improves the Isp by 5-10 seconds. The trend is reasonable because the average enthalpy of the gas is higher at the lower mass flow rate.

Both the experimental data and numerical results show an upward shift in efficiency at higher flow rate (Fig. 5). This trend is also a characteristic of other electrothermal devices, such as DC arcjets, and is generally attributed to frozen flow loss. However, for the pulsed arcjet running on helium propellant, the ionization losses are small. Further, since the efficiencies appear to be independent of area ratio, the increase in Isp at higher mass flow rates is probably due to energy loss effects in the capillary rather than viscous or ionization losses.

The Nozzle Flowfield

The five primary variables to be determined are the gas pressure, temperature, density, specific energy, and velocity \((p, T, \rho, e, u)\). The three conservation equations give the three flow properties, \(p, u\) and \(e\), and the two remaining flow properties, \(p\) and \(T\), are calculated from the ideal gas law and the Saha equation. Since the thruster is pulsed, the time-dependent terms are retained in the governing equations. The conservation equations of mass, momentum and energy for unsteady, quasi-1D flow are written in differential form.

\[
\frac{\partial p}{\partial t} = -\frac{1}{A} \frac{\partial}{\partial x} \left( \rho u A \right)
\]  
\[
\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - \frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{u^2}{2} \frac{4f}{D}
\]  
\[
\frac{\partial e}{\partial t} = -\frac{\partial e}{\partial x} - \frac{p \partial u}{\rho \partial x} - \frac{p u \partial A}{\rho A \partial x} + q_f + q_{\text{arc}}
\]  

The \(q_f\) term represents the conversion of kinetic energy into thermal energy due to friction, and \(q_{\text{arc}}\) is the heat transfer from the gas to the nozzle wall. Ohmic heating in the arc, \(Q_{\text{ARC}}\), and heat transfer in the capillary, \(Q_{\text{CAP}}\), are incorporated in the upstream boundary condition and do not appear in the nozzle equations. A friction factor is calculated based on a local Reynolds number, and heat transfer is determined based on a Nusselt number, as described in Ref. 3.

The MacCormack predictor-corrector algorithm has been used successfully by Anderson\(^4\) to study time-dependent chemically reacting flows in nozzles. In the calculation described here, the numerical solution for the nozzle begins at the entrance to the converging section, and the capillary is applied as a boundary condition, incorporated as a spatially-uniform pressure and temperature, \(p(t)\) and \(T(t)\). The grid geometry consists of a 45 degrees half-angle conical converging section, a throat region with a constant radius of curvature (2mm), and a 20-degree half-angle conical diverging section. For most calculations, 50 grid points are used, although sample calculations with up to 200 grid points were performed to verify grid independence. The size of each time step is set by numerical stability limitations and is calculated using a CFL (Courant-Friedrichs-Lewy) number equal to 0.8.

The capillary pressure and temperature are calculated by assuming a uniform state, uniform flow thermodynamic process. The governing equations are conservation of mass and energy,
The total mass and total energy of the propellant in the capillary are updated at each time step using these equations. The pressure and temperature are then calculated from the specific energy and density. The heat transfer from the propellant gas to the capillary walls, Q_{CAP}, is determined using a Nusselt number. For the boundary condition at the nozzle exit plane, the flow properties are extrapolated from interior points with the restriction that the flow is supersonic.

**Initial Condition**

Before the first pulse, the pressure in the capillary is established by steady choked flow at the given mass flow rate. When the first pulse occurs, the pulse energy and the initial propellant mass determine the increases in gas pressure and temperature. The pressure and temperature then decrease monotonically on a time scale that is slightly longer than the time between pulses. The second pulse occurs before conditions in the capillary return to the original starting conditions, such that energy addition from the second arc discharge starts at a higher initial temperature than the first discharge. The capillary flow conditions converge to a repeatable series after approximately 10-20 pulses, as shown in Fig. 6.

![Fig. 6. Numerical results for the initial propellant mass and gas temperature in the capillary during a typical startup series of pulses at 1800 pps.](image)

The average propellant mass in the capillary after the series of startup pulses is approximately one-half of the initial propellant mass, leading to higher peak temperatures than would be predicted if the starting series of pulses were not accounted for.

**Gasdynamic Results**

The numerical model was used to evaluate the relative importance of various design and operating parameters on pulsed arcjet performance. Results from the numerical model indicate that the performance is relatively insensitive to many design and operating parameters, including nozzle throat diameter, capillary length and diameter, pulse width, and peak power. Numerically, the circuit inductance was varied over two orders of magnitude (2.9 μH, 29 μH and 290 μH) with changes in the pulse width from 6.3 μsec to 50 μsec. The corresponding peak powers for these cases are 18.4 kW, 5.4 kW, and 1.7 kW. While the peak thrust decreases as the energy addition to the propellant is distributed over a longer time period, the total impulse is basically unchanged. Hence, high peak power levels are neither an advantage nor a disadvantage under these conditions.

The nozzle and capillary wall temperatures are by far the dominant variable affecting performance. The model predicts that efficiency can be increased to 62% at 401 seconds Isp, by improving the thermal design such that the wall temperatures reach 1400 K, typical for DC arcjets.

![Fig. 7. Typical numerical calculation for thrust, capillary pressure, and exhaust velocity during a single pulse.](image)

**Nozzle Parameters**

Pulsed arcjet performance is calculated with the numerical model for three nozzle area ratios (4, 20, and 230) over a range of specific energies. The specific energy is varied by setting the input power constant at 120 watts and changing the propellant flow rate. At constant power, the average specific energy varies roughly with the inverse of the throat Reynolds number, so that increasing the specific
energy can be expected to increase viscous effects in the nozzle. For specific energies less than approximately 10 MJ/kg, the Isp is highest for the 230:1 nozzle, with a gain of 10-15 seconds over the 4:1 nozzle. Viscous losses are relatively low in this regime, and the performance improves with larger area ratios due to the additional expansion of the gas. The specific impulses for the 20:1 and 230:1 nozzles are similar at low specific energy. This result is consistent with the steady state experimental results where the specific energies are less than 11 MJ/kg, and the Isp is similar for the two nozzles. The specific impulse for the 230:1 nozzle decreases rapidly due to viscous effects at specific energies above 10 MJ/kg, with the exit Mach number decreasing to unity at 18 MJ/kg. As shown in Fig. 8, the 20:1 nozzle maintains the best Isp between 11 and 33 MJ/kg.

Fig. 8. Specific impulse versus average specific energy for three nozzle area ratios.

**Arc Current and Arc Column Model**

The magnitude and shape of the current pulse affect the amount of energy transferred to the propellant gas compared to the energy lost in the external circuit. In addition, the shape of the current pulse is likely to be an important parameter for the long-term erosion characteristics of the electrodes and for capacitor life. On the other hand, the shape of the current pulse has only a minor effect on the gas dynamics of the flow because the arc discharge occurs on a time scale much shorter than the characteristic flow time.

**Analytical Model**

The time-development of the arc current is established by 1) the design of the pulse forming electrical circuit and 2) the characteristics of the arc resistance. Ignoring the diode for the moment, the circuit behaves similar to (but not exactly as) a simple LCR circuit, and the circuit equation can be written,

\[ V_b = \frac{1}{C} \int_0^t dt - (L_{\text{EXT}} + L_{\text{ARC}}) \frac{dI}{dt}, \quad (R_{\text{EXT}} + \frac{dI}{dt}) I - R_{\text{ARC}} I = 0 \quad (6) \]

The key solving the circuit equation is in determining the analytical form of the arc resistance. A number of models for predicting the resistance in a transient arc discharge can be found in the literature, as summarized by Engel. A model that appears to work well for the pulsed arcjet is to assume that the arc resistance varies inversely with the arc current, such that the voltage drop across the arc is constant during the pulse. The arc inductance, \( L_{\text{ARC}} \), is ~10 nH and can be neglected compared to the 2.9 \( \mu \)H external circuit inductance. Replacing the arc resistance, \( R_{\text{ARC}} \), with \( V_{\text{ARC}} / I \), the circuit equation can be solved analytically for the current. The equations for calculating the current have been developed by Robiscoe.

In the circuit with the diode, the arc current increases from zero to the peak current in approximately 1 \( \mu \)sec, and the voltage on the capacitor decreases from \( V_b \) to zero. During this time, current does not flow through the diode, and the circuit equation is identical to that for the circuit without the diode,

\[ i(t) = V_s \sqrt{\frac{C}{L_{\text{EXT}}}} \exp \left( \frac{-R_{\text{EXT}} t}{2 L_{\text{EXT}}} \right) \sin \left( \sqrt{\frac{1}{L_{\text{EXT}} C}} t \right) \quad (7) \]

where \( V_0 \) is equal to the breakdown voltage minus the arc voltage, \( V_b - V_{\text{ARC}} \). Approximately 10-20 percent of the initial stored energy is transferred to the propellant during the initial current rise, and the remaining energy is temporarily stored in the magnetic field of the circuit inductance, \( E = \frac{1}{2} L_{\text{EXT}} I^2 \). After the time of peak current, the capacitor is essentially removed from the circuit, and the current flows through the diode return loop. During the second half cycle, the circuit equation is,

\[ L_{\text{EXT}} \frac{dI}{dt} - V_{\text{ARC}} - V_{\text{DIODE}} - V_{\text{EXT}} = 0 \quad (8) \]

where \( V_{\text{ARC}} \) is the voltage drop across the arc, \( V_{\text{DIODE}} \) is the voltage drop in the diode (~2-3 volts), and \( V_{\text{EXT}} \) is the voltage drop associated with the external circuit resistance at high frequency (~20 m\( \Omega \)).

**Arc Current Model Validation**

In the analytical models, the breakdown voltage, arc voltage, and external resistance during the first half cycle are obtained from experimental data. The
breakdown voltage is measured directly, and the other two variables are obtained from curve fits to the current data. The calculated results for arc current are compared to data for a typical pulse in Fig. 9. The analytical model closely matches the experimental data with some deviations near the current peak and at the end of the pulse. The middle section of the current trace is linear, partially justifying the initial assumption of constant $V_{ARC}$.

![Fig. 9. Comparison of experimental arc current with the analytical solution, $V_{ARC} = \text{constant.}$](image)

The Arc Channel Model

The transient arc resistance can be estimated from Ohm's law and the experimental values for arc voltage and current. However, this is insufficient information to calculate the arc temperature because the arc radius is not known. The pulse energies in these experiments are too low to produce an arc column that fills the capillary entirely. It is well known that arcs tend to run at constant voltage, and these experimental results follow that condition closely. The Steenbeck minimum principle, or principle of minimum power, is a method for arc columns that fits most experimental data quite well. However, according to Raizer, this principle is not implied by the fundamental laws of physics, and further, this assumption is a supplementary principle that is not needed to calculate the behavior of the arc column. The approximation called the channel model, also suggested by Steenbeck, gives self-consistent results for an arc column and is used here.

In the channel approximation, the arc is assumed to have a constant temperature and electrical conductivity across the arc radius, $r_0$, which is less than the tube, or capillary, radius, $R$. Energy is transferred to the arc by ohmic heating and then is transported radially outward by heat conduction. The energy balance in the channel is given by

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \sigma(T)E^2 = 0$$  \hspace{1cm} (9)$$

where $E$ is the electric field. This equation is called the Elenbaas-Heller equation for steady arcs. Strictly speaking, the transient arc discharge requires a time-dependent term in the energy equation. However, if the arc radius is very small, the energy in the discharge is transported radially outward on a timescale that is rapid compared to the discharge time. Hence, using this equation 'as is' is essentially a quasi-steady approximation. An improvement to this analysis would include the time-dependent energy terms as well as energy losses due to radiation.

The set of arc column equations are given by Raizer, and these equations allow the arc column characteristics to be calculated for a specified arc current, $I$. The experimental data for a pulse are used to calculate the arc voltage. The calculated arc voltage (Fig. 10) is relatively constant and agrees well with the value obtained from the slope of the current trace. The arc resistance (Fig. 11) decreases to a minimum within 1 $\mu$sec to ~ 600 m$\Omega$ and then increases monotonically. The calculated arc channel radius ranges from 0.3 to 0.6 mm in the 1.25 mm diameter capillary, representing 6 to 23 percent of the total capillary volume. At higher pulse energies where the arc fills the capillary, a wall-confined arc model may need to be applied rather than the channel model equations.

![Fig. 10. Arc voltage calculated using the channel model and experimental data for arc current. The channel model agrees closely with the constant arc voltage approximation.](image)
Fig. 11. Arc resistance and channel radius from the channel model. The arc occupies 6 to 23 percent of the total capillary volume in the 2.5 mm diameter capillary.

**Summary and Conclusions**

The subject of this research is the experimental and numerical investigation of a pulsed arcjet operating on helium propellant at low power. The experimental work included the design and fabrication of thruster hardware and development of techniques to measure thruster performance and arc characteristics. A maximum $I_{sp}$ of 305 seconds was achieved at 38% efficiency. An important part of the experimental work is the development of the non-reversing pulse-forming electrical circuit, which may also have applications in other types of electric propulsion devices. Power losses in the storage capacitor can be large if the current is allowed to oscillate through the capacitor for a large number of cycles. However, the energy losses for a given circuit can be predicted with reasonable accuracy, and the addition of the diode to the circuit produced transfer efficiencies greater than 85% without current reversals in the capacitor.

The numerical work consisted primarily of a quasi-1D time-dependent gasdynamic model to calculate the specific impulse and thrust efficiency as well as the individual energy losses in the arcjet. The numerical results agreed reasonably well with the experimental data within the limitations of the quasi-1D formulation, and indicate that the efficiency with helium propellant can be increased to 62% at 401 seconds $I_{sp}$ by improving the thermal design. The primary energy loss mechanism is heat transfer from the gas to the capillary walls in the subsonic region. Wall temperature is a critical parameter for high performance, while the performance is relatively insensitive to other design variables.

For many operating conditions, the nozzle design is not critical, and the performance gains that can be obtained through optimization are small. An exception to this condition is for very low Reynolds numbers and low mass flow rates, where viscous losses decelerate the flow in large area ratio nozzles. The numerical results also indicate that there is an optimum value of the specific energy for a given nozzle at a particular input power. An unexpected conclusion from the numerical model is that average input power has a large effect on $I_{sp}$, while the influence of the peak power is small.


